MAT 101 (5. Habersiz Sınav) **1.Grup (8:30-10:20)** **27/12/2013**

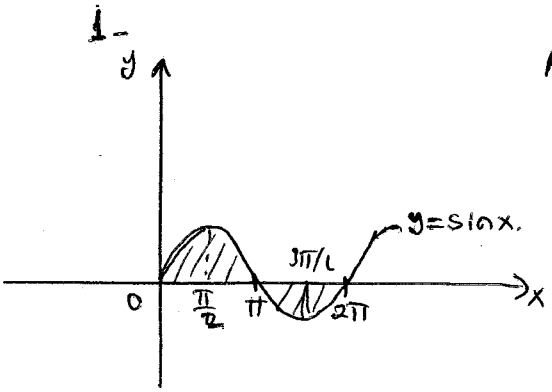
1- $y = \sin x$ eğrisi ve $y = 0$ doğrusu ile sınırlı bölgenin $x = 0$ ve $x = 2\pi$ arasında kalan kısmının alanını bulunuz.

2- $\int \frac{x}{\sqrt{4-x^2}}\,dx = ?$

SÜRE: 15dk. (10+10=20 puan)

ÇÖZÜM:

1-



$$A = \int_0^{2\pi} |\sin x|\,dx$$

$$= \int_0^{\pi} \sin x\,dx - \int_{\pi}^{2\pi} \sin x\,dx$$

$$= -(\cos x)\Big|_0^{\pi} + (\cos x)\Big|_{\pi}^{2\pi}$$

$$= -\underbrace{\cos\pi}_{-1} + \underbrace{\cos 0}_{1} + \underbrace{\cos 2\pi}_{1} - \underbrace{\cos\pi}_{-1}$$

$$= 1 + 1 + 1 + 1 = 4 \text{ br}^2.$$

2- $\left(u = 4 - x^2 \Rightarrow du = -2x\,dx \Rightarrow -\frac{du}{2} = x\,dx\right)$

$$\int \frac{x}{\sqrt{4-x^2}}\,dx = -\frac{1}{2}\int \frac{du}{\sqrt{u}} = -\frac{1}{2}\int u^{-1/2}\,du = -\frac{1}{2}\cdot 2\,u^{1/2} + c = -\sqrt{u} + c.$$

$$= -\sqrt{4-x^2} + c$$

MAT 101 (5. Habersiz Sınav) **2.Grup (10:30-12:20)** **27/12/2013**

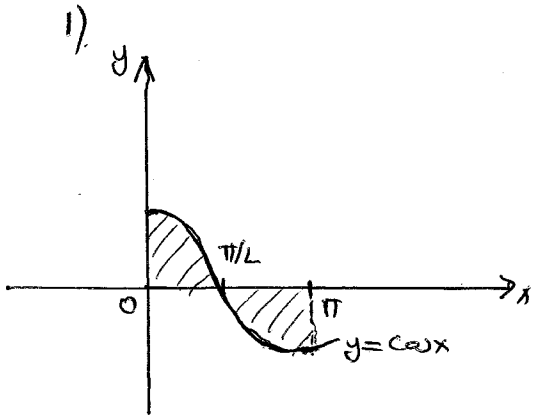
1- $y = \cos x$ eğrisi ve $y = 0$ doğrusu ile sınırlı bölgenin $x = 0$ ve $x = \pi$ arasında kalan kısmının alanını bulunuz.

2- $\int \frac{dx}{(9-x^2)^{3/2}} = ?$

SÜRE: 15dk. (10+10=20 puan)

ÇÖZÜM:

1)



$$A = \int_0^{\pi} |\cos x|\,dx = \int_0^{\pi/2} \cos x\,dx - \int_{\pi/2}^{\pi} \cos x\,dx$$

$$= (\sin x)\Big|_0^{\pi/2} - (\sin x)\Big|_{\pi/2}^{\pi}$$

$$= \underbrace{\sin\tfrac{\pi}{2}}_{1} - \underbrace{\sin 0}_{0} - \underbrace{\sin\pi}_{0} + \underbrace{\sin\tfrac{\pi}{2}}_{1}$$

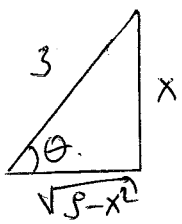
$$= 1 - 0 - 0 + 1 = 2 \text{ br}^2.$$

2) $\left(x = 3\sin\theta \Rightarrow dx = 3\cos\theta\,d\theta.\right)$

$$\int \frac{dx}{(9-x^2)^{3/2}} = \int \frac{3\cos\theta\,d\theta}{(9-9\sin^2\theta)^{3/2}} = \int \frac{3\cos\theta\,d\theta}{3^3(\underbrace{1-\sin^2\theta}_{\cos^2\theta})^{3/2}}$$

$$= \int \frac{\cos\theta\,d\theta}{9\cdot\cos^3\theta} = \frac{1}{9}\int \frac{1}{\cos^2\theta}\,d\theta$$

$$= \frac{1}{9}\int \sec^2\theta\,d\theta = \frac{1}{9}\tan\theta + c.$$



$$= \frac{1}{9}\frac{x^2}{\sqrt{9-x^2}} + c.$$

MAT 101 (5. Habersiz Sınav) **3.Grup (13:30-15:20)** **2712/2013**

1- $y = e^x$ eğrisi, $x = 0$ ve $y = 2$ doğruları ile sınırlı bölgenin alanını bulunuz.

2- $\int \frac{1}{(4+x^2)^{3/2}} dx = ?$

SÜRE: 15dk. (10+10=20 puan)

ÇÖZÜM:

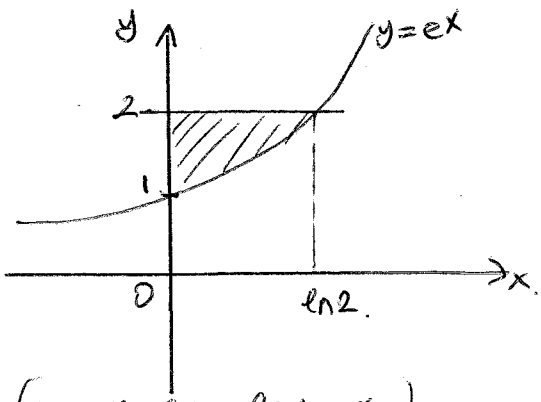
1)

$2 = e^x \Leftrightarrow \ln 2 = x$

$$A = \int_0^{\ln 2} (2 - e^x)\, dx$$

$$= (2x - e^x)\Big|_0^{\ln 2} = 2\ln 2 - 2 - e^0$$

$$= 2\ln 2 - 3 \text{ br}^2$$



$(y = e^x \Leftrightarrow \ln y = x)$

II. yol:

$$A = \int_1^2 \ln y\, dy = (y \cdot \ln y - y)\Big|_1^2$$

$$= 2\ln 2 - 2 - 1\ln 1 + 1$$

$$= 2\ln 2 - 3 \text{ br}^2.$$

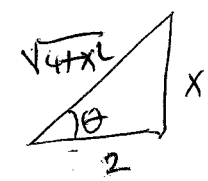
2). $x = 2\tan\theta \Rightarrow dx = 2\sec^2\theta\, d\theta.$

$$\int \frac{2\sec^2\theta\, d\theta}{(4+4\tan^2\theta)^{3/2}} = \int \frac{2\sec^2\theta\, d\theta}{2^3(1+\tan^2\theta)^{3/2}} = \int \frac{\sec^2\theta}{4\sec^3\theta} = \frac{1}{4}\int \cos\theta\, d\theta$$

($1+\tan^2\theta = \sec^2\theta$)

$$= \frac{1}{4}\sin\theta + c$$

$$= \frac{1}{4}\,\frac{x}{\sqrt{4+x^2}} + c.$$



$\frac{x}{2} = \tan\theta.$

MAT 101 (5. Habersiz Sınav) **4.Grup (15:30-17:20)** **27/12/2013**

1- $y=\sqrt{x}$ eğrisi ve $y=x$ doğrusu ile sınırlı bölgenin alanını bulunuz.

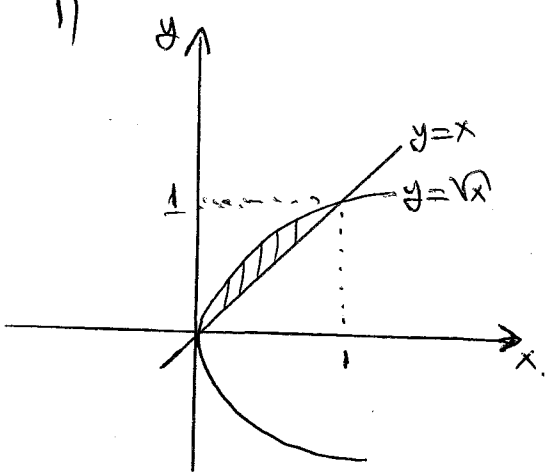
2- $\int \frac{\sqrt{x^2-4}}{x}dx = ?$

SÜRE: 15dk. (10+10=20 puan)

ÇÖZÜM: $\sqrt{x}=x \Leftrightarrow x=x^2 \Leftrightarrow x^2-x=0 \Leftrightarrow x(x-1)=0$

$\Leftrightarrow x=0$ veya $x=1$

1)



$$A=\int_0^1 (\sqrt{x}-x)\,dx$$

$$=\left(\frac{2}{3}x^{3/2}-\frac{x^2}{2}\right)\Big|_0^1 = \frac{2}{3}-\frac{1}{2}=\frac{4-3}{6}=\frac{1}{6}\ br^2.$$

II. yol.

$$A=\int_0^1 (y-y^2)\,dy = \left(\frac{y^2}{2}-\frac{y^3}{3}\right)\Big|_0^1$$

$$=\frac{1}{2}-\frac{1}{3}=\frac{3-2}{6}=\frac{1}{6}\ br^2$$

2) $x=2\sec\theta \Rightarrow dx=2\sec\theta\tan\theta\,d\theta.$

$$\int \frac{\sqrt{x^2-4}}{x}dx = \int \frac{\sqrt{4\sec^2\theta-4}}{2\sec\theta}\cdot 2\sec\theta\tan\theta\,d\theta$$

$$=\int 2\cdot\sqrt{\sec^2\theta-1}\,\tan\theta\,d\theta = 2\int \tan^2\theta\,d\theta = 2\int(\sec^2\theta-1)\,d\theta.$$

$$=2(\tan\theta-\theta)+C = 2\,\frac{\sqrt{x^2-4}}{2}+\sec^{-1}\frac{x}{2}+C.$$

bulunur.

$x=2\sec\theta$

$\Rightarrow \frac{x}{2}=\sec\theta=\frac{1}{\cos\theta}$

$\Leftrightarrow \cos\theta=\frac{2}{x}$ olur