MATHEMATICAL MODELLING OF DIRECT CURRENT MOTORS

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ABSTRACT

Universities have an important role on the development of a country, not only providing educated staff but also with their scientific studies and cooperation with the industry.

In this study, the mathematical modeling of permanent magnetic motors that are widely used in industry, were taken as sample.

KEY WORDS: Direct Current Motors, Automatic Velocity Control, Mathematical Modeling

INTRODUCTION

DC motors are increasingly being used. Since there is a linear relationship between the input voltage and rotation in DC motors, it is possible to control precisely the speed by computer, microprocessors or even analog circuits.

If we examine the utilization areas of DC motors, printers, disk drivers, numeric controlled machinery, photocopy devices, plotters, medical devices, robots and packaging machines, textile machinery, airplane navigation control systems, flow rate control valves and some lifting machines can be mentioned as samples.

When, a speed control with various algorithms (PI-Proportion-Integral and PID-Proportion-Integral-Differential Control) is applied to DC motors, to forecast the continuous regime fault on the speed – time curve and control faults depend on control parameters, simulation models based on mathematical models are generally used.

In this study, a different method was used (U input was considered as I current) and the coefficients obtained by using the real working principles of the motor, a mathematical model of the motor is derived.

THE STUDIES ON DC MOTOR CONTROLLING

Bulca and Turkay performed a comparison for the PID control of a DC motor controlled by a microprocessor. In their study, the basic and developed PID speed algorithms were used onto the control of a microprocessor controlled system and it was compared experimentally.

Based on this, it is confirmed that, PD position algorithm which was developed for position controlling and basic PI algorithm for speed control have fast and accurate responses. In the results of speed control, while basic PI algorithm provides fast and regular extinguishing responses, yet, the developed PI speed algorithm provides worse responses than the basic version and this is stated as an unexpected result.

Laupolos Larybakas, put forward a phase locked control schema for DC motors that provide fastly speed adaptation capacity to input command with different loads. An operating limit increasing assistant frequency provider was added to the system and a load irregularity removing MRAC Type load control was defined.

Sebakhy designed an adaptive controller for DC motors working under different load situations. The controller is defined as, a self tuning version of a control algorithm which minimizes the total integration faults of speed on the frequency regulation time of limited free speed together with reference input and control energy.

With simulation results, it is proved that this method rejects the load irregularities and creates optimal performance. In this article, it is stated that, when the parameters are unknown or in an infinite variations, self-adjusting version of the dead point algorithms are used to control the output.

Sunha and Bailey applied phase locked cycle onto the experiment mechanism. Based on the test results, they put forward that high performance could be obtained with the cheap commercial integrates carrying integrated circuit chips. Although, the speed encoder of the system produces square impulses and reference frequencies were square, and, since the system was neither digital nor there was a phase locked cycle, the precision was %0.002 which is sufficient the speed control of the DC motor was stated in the article. Additionally, the more precision will be obtained by increasing the precision of encoder, and by removing the noise on the frequency using a proper impulse forming mechanism between the photo-transistor and phase detector.

Maloney and Alvarado, on the other hand, define a motor speed controller using digital techniques are being used to measure, compare and adjust the motor speed in their article. They claim that the digital method they used increases the productivity of speed bodies by eliminating the measuring nonlinearities. It is also told that, the continuous

regime fault of the prototype is less than %2 of the maximum speed and at the developed design this fault decreases under %1.

DERIVATION OF MATHEMATİCAL MODEL

The symbols used in the article are given down below;

$\phi(t)$	Magnetic Flow
e_a	Rotary Voltage
e_f	Area Voltage
θ_m	Motor Rotation Angle
G(s)	Transfer Function
x_i	Condition Variables
K_m	Motor Gain
τ_m	Time Constant
$M_m(t)$	Motor momentum

As it is known the DC motors are classified as Alternative Magnetic Flux Motors and Constant Magnetic Flux Motors. Because of the stator current of alternative magnetic flux motors are alternating, the mathematical model of these motors is 4^{th} degree.

Today, since the motors widely used in the industries have natural magnets, the stator current is constant and their mathematical models step down to 3rd degree. Hence the mathematical model used in speed control is derived for these types of motors.

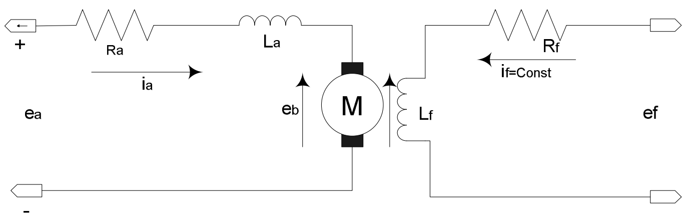

Figure 1. Model of Constant Field Wind DC motor

Magnetic Flow $\phi(t)$;

(1) $$\phi(t) = K_f . i_f(t) = K_f . I_f \quad \text{(=Constant)}$$

Field Voltage

(2) $$e_f(t) = R_f . i_f + L_f . \frac{di_f}{dt}$$

Since i_f is constant in DC motors, the area voltage is also constant and has no effect.

Rotor Voltage is

(3) $$e_a(t) = R_a.i_a + L_a.\frac{di_a}{dt} + e_b$$

Reverse electromotor voltage is indirectly proportional with motor speed.

(4) $$e_b(t) = K_b.\frac{d\theta_m}{dt}$$

Motor moment is

(5) $$M_m(t) = K_m.\phi(t).i_a(t) = K_m.K_f.I_f.i_a(t) = k.i_a(t)$$

At the same time momentum can be obtained as,

(6) $$M_m(t) = J.\frac{d^2\theta_m}{dt^2} + B.\frac{d\theta_m}{dt}$$

If we reorganize these equations into matrix form we receive the equations of (7) and (8)

(7) $$\frac{di_a(t)}{dt} = \frac{1}{L_a}.e(t) - \frac{R_a}{L_a}.i(t) - \frac{K_b}{L_a}.\frac{d\theta_m}{dt}$$

(8) $$\frac{d^2\theta_m(t)}{dt^2} = -\frac{B}{J}.\frac{d\theta_m(t)}{dt} + \frac{k}{J}.i_a(t)$$

Here, it is possible to see the position variables are stator current, motor position and motor speed. The position variables are:

$$x_1 = \theta_m \quad , \quad x_2 = \dot{\theta} = \omega_m \quad , \quad x_3 = i_a \quad , \quad u = e_a$$

From this point

(9) $$\dot{x}_1 = x_2$$

(10) $$\dot{x}_2 = -\frac{B}{J}.x_2 + \frac{k}{J}.x_3$$

(11) $$\dot{x}_3 = -\frac{K_b}{L_a}.x_2 - \frac{R_a}{L_a}.x_3 + \frac{1}{L_a}.u$$

If this system is expressed in matrix (canonical) form,

$$(\dot{X} = A.X + B.U)$$

(12)
$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -\frac{B}{J} & \frac{k}{J} \\ 0 & -\frac{K}{L_a} & -\frac{R}{L_a} \end{bmatrix} . \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} .u$$

From (9), (10) and (11) equations, we obtain a computer diagram as in Figure-2

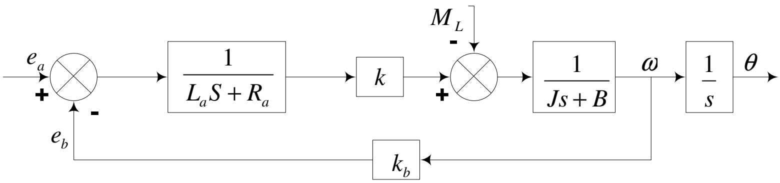

Figure 2. Constant Area Wind DC motor block diagram

If the block diagram of the system considered as $M_L = 0$, the close loop transfer function will be:

(13)
$$G(s) = \frac{\Omega(s)}{E(s)} = \frac{k}{(L_a s + R_a).(Js + B) + k.K_b}$$

(14) Mechanic time constant of the system: $\tau_m = \frac{J}{B}$

(15) Electrical time constant: $\tau_e = \frac{R_a}{L}$

If we put (14) and (15) into (13):

(16) $$G(s)=\frac{\Omega(s)}{E(s)}=\frac{k}{R.B.(\tau_e s+1).(\tau_m s+1)+k.K_b}$$

can be written. In this mathematical modeling, when $\tau_m >> \tau_e$, mathematical model becomes 2nd degree. Since $\tau_m > 10.\tau_e$, Ra / L_a = 0

Hence, the decreased system's transfer function becomes:

(17) $$G(s)=\frac{\Omega(s)}{E(s)}=\frac{k}{R_a.(Js+B)+k.K_b}$$

The block diagram of this system is given at Fifure-3.

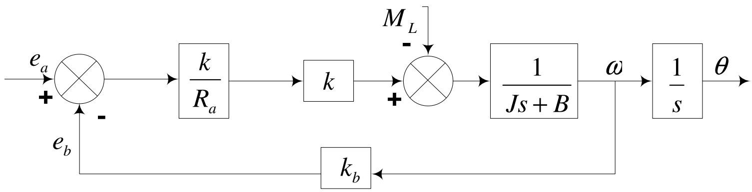

Figure 3. Decreased Block Diagram

To get this model of system, a different method was used. In this method, the (u) input becomes (i) current. It is possible to write down the momentum by adding reverse electromotor voltage instead of this. Hence, the momentum:

(18) $$M = k.I = a.u - b.\dot{\theta}_m$$

a and b motor constants. To write down the momentum as in this form provides receiving a and b from the motor characteristics. As a result the mathematical model:

(19) $$M = J.\ddot{\theta} + B.\dot{\theta} = a.u - b.\dot{\theta}_m$$

When we reorganize this (19 no) equation,

(20) $$J.\ddot{\theta} + (B+b).\dot{\theta} = a.u$$

is obtained. The position constants are

(21) $x_1 = \theta_m$, $x_2 = \dot{\theta}_m = \omega_m$, $\dot{x}_1 = x_2$,

$$\dot{x}_2 = -\frac{1}{\tau_m}.x_2 + K_m.u$$

(22) And, $\tau_m = \dfrac{J}{B+b}$ (mathematical time constant)

(23) $K_m = \dfrac{a}{J}$ (Motor gain)

Closed loop transfer function of the system:

(24) $G(s) = \dfrac{K_m}{s.(\tau_m s + 1)}$ is obtained.

When the system is written as $\dot{X} = A.X + B.U$

(25)
$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & -\dfrac{B+b}{J} \end{bmatrix} . \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \dfrac{a}{J} \end{bmatrix} .u$$

A matrix mathematic model of DC motor is obtained

(a) and (b) constants derived from equation (18) are calculated from the motor characteristics given down below. There is a correlation between the rpm and momentum in DC motors as shown in the Figure-4 for a definite rotary voltage level.

For $M = a.u - b.\omega$, $\omega = 0$,

$M = M_{\max} = a.u_{\max}$ is obtained. Hence,

(26) Nominal Speed Momentum calculated as:

$$\omega_{nom} = \frac{2.\pi.n_{nom}}{60} \quad (rad/s)$$

$P = M.\omega_{nom}$ If we take M from this equation, we obtain:

$$M = \frac{P}{\omega_{nom}} \quad (N.m)$$

$$a = \frac{M_{\max}}{U_{\max}} \quad (Nm/V)$$

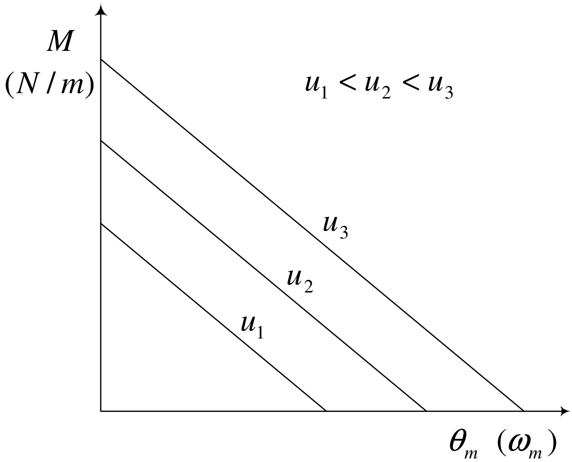

Figure 4. Speed – Moment Curves of DC motors

Here b constant can be found from $M = M_{\max} - b.\omega_{nom}$ equation
And

(27) $$b = \frac{M_{\max} - M}{\omega_{nom}} \quad (N.m.s/V)$$ is received. Here,

P= motor power (W)
M_{max} = maximum motor momentum (N.m)
U_{max} = maximum motor input voltage (V)
n_{nom} = nominal motor rpm is expressed as (d/d)

CONCLUSION

In this study, the results of simulation studies performed by using the mathematical model derived by using the constants obtained from the characteristic curves of DC motors will be helpful to produce nearest results to the results obtained from the real model.

REFERENCES

1. KUO, Benjamin C.& TAL, Jacob. Incremental Motion , "DC Motors and Control System ", Vol 1, SRL Publishing Company , 1978
2. KUO Kuo, Benjamin C., "Automatik Control System ", pp. 166-176 Pretice Hall International, Inc., 1987
3. N.Özdaş -T.Dinibütün-A.Kuzucu - "Otomatik Kontrol ", 1988 , İ.T.Ü. Matbaası - Gümüşsuyu, 1988
4. E.A.Parr -" Endüstriyel Kontrol ", Cilt 1, Evren Ofset Basım Sanayii ve Ticaret A.Ş., 1994
5. Feedback Control of Dynamics Systems – Franklin, Powell, Emami-Naeimi Addision Wesley, 2002